POS 3713 Spring 2000
Review Problems for Midterm Exam

The midterm exam is scheduled on Wednesday, March 15th. I will set up a review time on Monday or Tuesday evening of that week.

1. An instructor gives his class an examination that, as he knows from years of experience, yields a mean (μ) of 78 and a standard deviation (σ) of 3.5. His present class of 22 students obtains a mean of 82. Is he correct in assuming that this is a superior class (i.e., this class is not part of the general population)? Assume 95% confidence, or $\alpha = 0.05$.

Solution: $H_o: \mu = 78$
$H_1: \mu > 78$ (because we are testing if the present class is superior)

We use Z because σ is known.

$$Z = \frac{X - \mu}{\sigma/\sqrt{N}} = \frac{82 - 78}{3.5/\sqrt{22}} = 5.36$$

Since 5.36 > 2.32 (critical Z), we reject H_o and conclude that this is a superior class (in other words it is not part of the general population).

2. In the year 2058, the Federation of Planets is interested in assessing the average number of years served on each of its starships. It believes that the average number of years served on the Starship Enterprise is greater than the average among the rest of its galaxy starships because the Enterprise attracts good personnel and is the best ship in the fleet. The Federation has calculated the sampling distribution of the mean years served on all starships. They find that μ=7.5 and σ=2.4. The Federation's survey team finds that the average number of years served on the Enterprise among 100 of its crew members (N=100) is 9.5. Assume 95% confidence, or $\alpha = 0.05$.

a. Calculate the 95% confidence interval for the mean number of years served on the Starship Enterprise. How do you interpret this interval?

Solution: $\mu = X \pm Z_{\alpha/2} \cdot \sigma/\sqrt{N} = 9.5 \pm 1.96(2.4/\sqrt{100}) = 9.5 \pm .4704$

Interpretation: We are 95% confident that the mean number of years served on the Starship Enterprise is between 9.03 and 9.97 years.

b. Can the Federation of Planets be 95% ($\alpha = 0.05$) certain that the Enterprise is part of the Federation population of starships? In other words, is the average number of years served on the Starship Enterprise greater than the average among the rest of its galaxy starships?

Solution: In this case, we are speculating that the mean number of years served on the Enterprise is greater than for other Federation starships, so we should employ a one tailed hypothesis test. We use Z because σ is known.

H_o: $\mu = 7.5$
H_1: $\mu > 7.5$

$$Z = \frac{X - \mu}{\sigma/\sqrt{N}} = \frac{9.5 - 7.5}{2.4/\sqrt{100}} = 8.33$$

Since 8.33 > 1.64 (critical Z), we reject the null hypothesis and conclude that the mean number of years served on the Starship Enterprise is greater than the mean number of years served on all Federation starships. In other words, the Federation of Planets cannot be sure that the Enterprise is part of their population, and it is not reasonable to assume that this sample was drawn from the entire population.

3. A random sample of 26 local political science graduates scored an average of 458 on the GRE advanced political science test with a standard deviation of 20. Is this significantly different from the national average? ($\mu = 440$). Assume 95% confidence, or $\alpha = 0.05$. (Hint: this is a small sample, and σ is unknown)

Solution: H_o: $\mu = 440$
H_1: $\mu \neq 440$

We use t because N is small and σ is unknown.

$$t = \frac{X - \mu}{s/\sqrt{N-1}} = \frac{458 - 440}{20/\sqrt{26-1}} = 18/4 = 4.5$$

Since 4.5 > 2.06 (critical t for 25 degrees of freedom (df=N-1)), we reject H_o and conclude that the local average is significantly different from the national average.

4. Soon after he took office in 1963, 160 out of 200 Americans sampled approved of President Johnson. With growing disillusionment over his Vietnam policy, by 1968 he was approved by only 70 out of a sample of 200 Americans. What is the 95% confidence interval for the percentage of all Americans who approved of Johnson in 1963? In 1968? (Hint: use the confidence interval formula for proportions) How do you interpret these confidence intervals?

Solution: 1963: $\pi = P_s \pm Z_{\alpha/2}\cdot\sqrt{[P_u(1-P_u)]/N} = .8 \pm 1.96\cdot\sqrt{[.5(1-.5)]/200} = .8 \pm .069$
Interpretation: We are 95% confident that 73.1% to 86.9% of Americans approved of Johnson in 1963.

1968: $\pi = P_s \pm Z_{\alpha/2}\cdot\sqrt{[P_u(1-P_u)]/N} = .35 \pm 1.96\cdot\sqrt{[.5(1-.5)]/200} = .35 \pm .069$
Interpretation: We are 95% confident that 28.1% to 41.9% of Americans approved of Johnson in 1968.

5. Calculate the index of qualitative variation (IQV) for party identification using the 1994 National Election Study data presented below. What does the IQV tell you about the dispersion of party identification in the United States?

Party Affiliation	Frequency
Democrat	612
Independent	513
Republican	544

Solution: We calculate the IQV as follows: $IQV = \frac{k\,(N^2 - \Sigma f^2)}{N^2\,(k - 1)}$

$$IQV = \frac{3(1669^2 - (612^2 + 513^2 + 544^2))}{1669^2\,(3-1)} = \frac{3(2785561 - 933649)}{5571122} = \frac{5555736}{5571122}$$

$IQV = 0.997$

Interpretation: The IQV is extremely close to 1, indicating that party identification is quite dispersed. In other words, each party is likely to attract approximately the same number of people (recall that IQV = 1 when the frequencies in each category are equal).

6. An FSU professor administers a standard IQ test in POS 3713. The sample mean is equal to 90 and the sample standard deviation is 10. Assume that the variable is normally distributed.

a. Sara obtained a score of 110 on the test. What *percent* of cases fall between Sara's score and the mean?

Solution: Here we are comparing a single score in a distribution to the mean, so we calculate a Z score.

$$Z = \frac{X - \bar{X}}{s} = \frac{110 - 90}{10} = 2$$

Area between mean and Z = 2 is .4772. 47.72% of cases fall between Sara's score and the mean.

b. What is Sara's *percentile rank* in the population? How do you interpret this percentage?

Solution: The percentile rank refers to the percentage of people who score below a particular person. Thus we add .5 to .4772 (the area between Sara's score and the mean) to obtain a total of .9772. Sara's percentile rank is the 97.72%, which means that 97.72% of people have a lower IQ than Sara (she is really smart!).

c. What *percent* of cases fall between a score of 105 and 85?

Solution: We need to calculate two Z scores in this case, one for 105 and one for 85.
For 105:

$$Z = \frac{X - \bar{X}}{S} = \frac{105 - 90}{10} = 1.5$$

Area between Z = 1.5 and mean is .4332

For 85:

$$Z = \frac{X - \bar{X}}{S} = \frac{85 - 90}{10} = -0.5$$

Area between Z = -0.5 and mean is .1915

.4332 + .1915 = .6247
62.47% of cases fall between a score of 105 and 85.

7. Calculate the mean, median, mode, variance, and standard deviation for the following golf scores. Think about how to interpret each of these statistics substantively.

   70 72 68 65 71 75 66 67 65 70

Mean = (70 + 72 + 68 + 65 + 71 + 75 + 66 + 67 + 65 + 70)/10 = 689/10 = 68.9

Median: arrange values in ascending order
65 65 66 67 68 70 70 71 72 75
Since N is even, we take the average of the 2 middle values: (68 + 70)/2 = 69

Mode: most frequently occurring value; in this case we have 2 modes: 65 and 70.

$$\text{Variance} = s^2 = \frac{\Sigma (X_i - \text{Xbar})^2}{N - 1}$$

$s^2 = [(70 - 68.9)^2 + (72 - 68.9)^2 + (68 - 68.9)^2 + (65 - 68.9)^2 + (71 - 68.9)^2 + (75 - 68.9)^2 + (66 - 68.9)^2 + (67 - 68.9)^2 + (65 - 68.9)^2 + (70 - 68.9)^2]/(10 - 1)$
$s^2 = 10.77$

Standard deviation = $s = \sqrt{s^2} = \sqrt{10.77} = 3.28$
Interpretation: On average, this golfer scores 3.28 points above or below his/her mean score of 68.9.

8. Refer to the following theoretical hypothesis and contigency table (cross-tabulation) to answer the questions in the space below.

Theoretical Hypothesis: Women are more likely to identify with the Democratic party because it is more sensitive to issues that women care about, such as education, abortion (a pro-choice position), and family issues (such as maternal leave).

Data: Survey data from the 1994 National Election Study (N=1795, but some cases have been dropped for this analysis)

		Party Identification		
Sex	Republican	Independent	Democrat	Total
Male	273 (34.7%)	269 (34.2%)	244 (31%)	786 (47.1%)
Female	271 (30.7%)	244 (27.6%)	368 (41.7%)	883 (52.9%)
Total	544 (32.6%)	513 (30.7%)	612 (36.7%)	1669 (100%)

1. What is the independent variable for this theory? What is the dependent variable?

IV: sex, DV: party identification

2. What *percentage* of the total sample is male?

47.1%

3. What *proportion* of the total sample is Democratic?

612/1669 = .367

4. What *percentage* of respondents are female Independents?

244/1669 = .146 or 14.6%

5. How many females in this sample identify with the Republican party (i.e., what is the frequency)?

271

6. Does the table provide any evidence to support the theoretical hypothesis? Justify your answer using the information presented.

Yes, the table does provide evidence to support the hypothesis that women are more likely to identify with the Democratic party. The percentage of women identifying Democratic is 41.7%, which is much higher than the percentage of women identifying with the Republican party (30.7%) or declaring themselves Independent (27.6%).